MATHS HELP

PREFIXES

Electrical power is measured in watts, often abbreviated to W. In many cases, we need to talk about large numbers of watts. For example; a washing machine might have a power of 3000 watts.

When we use large numbers, it can be more convenient to change the size of the unit being used. In this case, when we wish to talk in thousands, we put 1000 watts together and call it a **kilo**watt, abbreviated to kW.

So the washing machine has a power of 3 kW. **Kilo** is known as a **prefix** – it is fixed on to the front of the word and multiplies the size by 1000.

There are other prefixes. The four main ones are:

Prefix	Multiplies	Abbreviation
tera-	1 000 000 000 000	T
giga-	1 000 000 000	G
mega-	1 000 000	M
kilo-	1 000	k

So 4000 watts = 4 kW and 5 000 000 ohms = 5 MΩ etc.

EXTENSIONS

You can also use prefixes for quantities **less** than 1. For example,

1 milliwatt (mW) = 1/1000 watts.
1 centimetre = 1/100 metre.

Prefix	Multiplies	Abbreviation
deci-	1/10	d
centi-	1/100	c
milli-	1/1000	m
micro-	1/1 000 000	μ

Write the following as simply as possible, using the table above.

(a) 1/1000 metre.

(b) 5/1 000 000 second.

(c) 1/200 litre.

PROBLEMS

1. Write as simply as possible, using a prefix abbreviation:

   (a) 7000 watts

   (b) 8 000 000 volts

   (c) 5500 ohms

   (d) 4 000 000 000 watts

2. Work out, and give the answer as simply as possible

   (a) 2 kΩ + 5 kΩ

   (b) 3 kΩ + 400 Ω

   (c) 4.7 kΩ + 15 kΩ

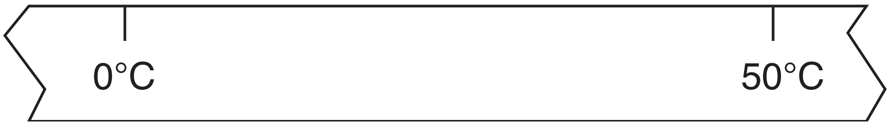
   (d) 4.7 kW + 550 W

   (e) 7 kV + 6.5 MV

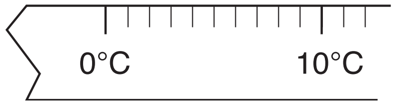
   (f) 5.3 kΩ + 500 Ω + 15.2 kΩ

MEASURES AND ERRORS

You are making a thermometer scale. It is a straight line scale, and all the divisions on it must be equally spaced. The positions of the lowest and highest temperatures, (0°C and 50°C) are 10.5 cm or 105 mm apart.

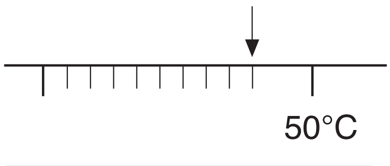


So, a change in temperature of 50°C is shown by the length 105 mm. This means a change of 1°C is shown by the length 105/50 = 2.1 mm. As it is difficult to measure 2.1 mm, you might decide to use 2 mm which is close.



Laying your ruler along the scale and, starting at 0, you make a mark every 2 mm.

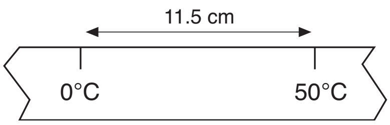
Finally, you would make the 50th mark as shown but it is not where it should be at the 50°C mark.



So what went wrong? Fifty spaces at 2 mm each is 50×2 = 100 mm but 105 mm were actually needed. Though 2 mm seems close to 2.1 mm, when used several times over it can make a big error in the end. The total error is 105 - 100 = 5 mm too small.

PROBLEMS

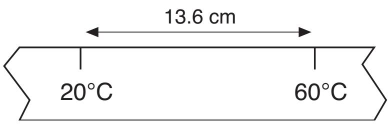
1. Sara is making a thermometer to measure from 0°C to 50°C. On her scale the distance between the two is 11.5 cm.



   (a) What should the distance be between each degree?

   (b) Sara uses 2 mm spacing to mark the degrees. What will her total error be at 50°C?

2. Mark is making a thermometer to show 20°C to 60°C. The measured distance between those points is 13.6 cm.



   (a) How many degrees does Mark's thermometer show?

   (b) What should the measured distance be for each degree?

   (c) He marks the degree spaces starting from 20°C. What will his total error be if he uses

   (i) 3 mm for each degree?
   (ii) 4 mm for each degree?

3. Rohan is making an electronic weighing machine to weigh between 100 and 800 grams only. The scale is a straight line and, between those two points, the distance is 11.3 cm. Rohan wants to put a mark on the scale at every 20 grams.

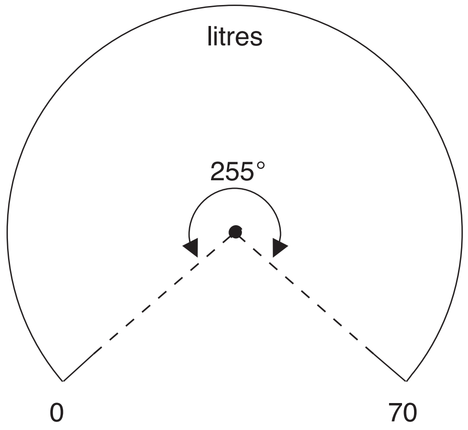
   (a) What distance is needed between each mark?

   (b) Working to the nearest millimetre, what error will Rohan have at the end?

   (c) If Rohan measured the marks in from both ends, what would his error be when he reached the middle?

DIVIDING AN ARC AND CALIBRATION

Our problem here is to calibrate a rotary control knob. For example, suppose you want to read the volume of liquid in a tank in litres. The lowest value is 0 litres and the largest value 70 litres. The knob turns through 255˚.



You want to read every 5 litres on the scale; 0, 5, 10, 15, ..., 65, 70. Since 70 litres is represented by an angle of 255˚, 1 litre is represented by

$$\frac{255°}{70} = 3.6428°$$

and 5 litres is represented by

$$5 \times \frac{255°}{70} = 18.214°$$

which is 18° to the nearest degree.

You can continue in this way:

10 litres needs an angle of

$$10 \times \frac{255°}{70} = 36.428°$$

which is 36° to the nearest degree.

15 litres needs an angle of

$$15 \times \frac{255°}{70} = 54.642°$$

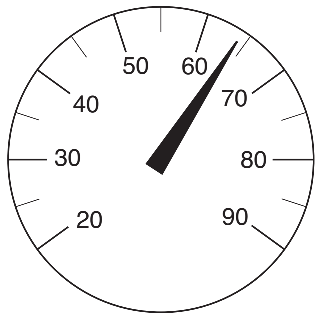
which is 55° to the nearest degree.

PROBLEMS

1. Copy and complete the table below to give all the required angles:

Reading (litres)	**Angle** (nearest degree)
0	0°
5	18°
10	36°
20	55°
25	-
30	-
35	-
40	-
45	-
50	-
55	-
60	-
65	-
70	255°

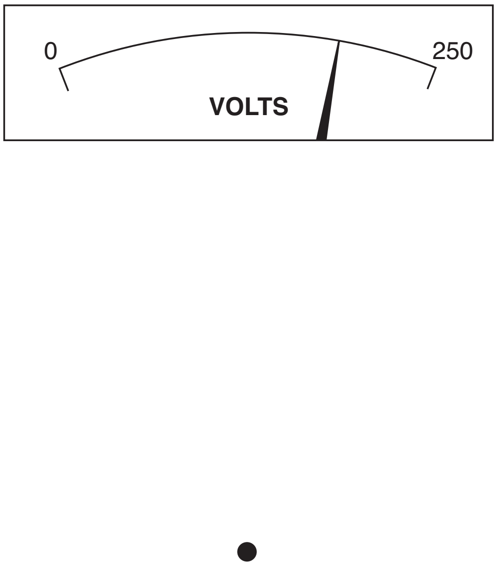
2. Using a circle of radius 4 cm, calibrate a 255° sector of a circle to show 0 to 70 litres in 5 litre divisions.
3. A completed calibration from 20 to 90 is shown in the diagram below. Estimate the reading shown by the pointer.



4. A similar dial has to be calibrated from 0 to 130 litres, marked off in 10 litre intervals. The total angle to be used is 290°. Complete a table of angle calculations and draw the diagram.

EXTENSION

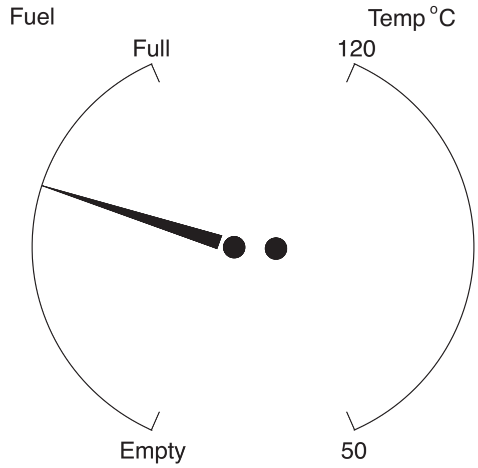
Suppose you use 18° for each 5 litre division in the first problem. Would your calibration be accurate enough?



1. Complete the marking of the voltmeter scale to show 50, 100, 150 and 200 volts.

What voltage is being shown by the pointer?

_______ volts



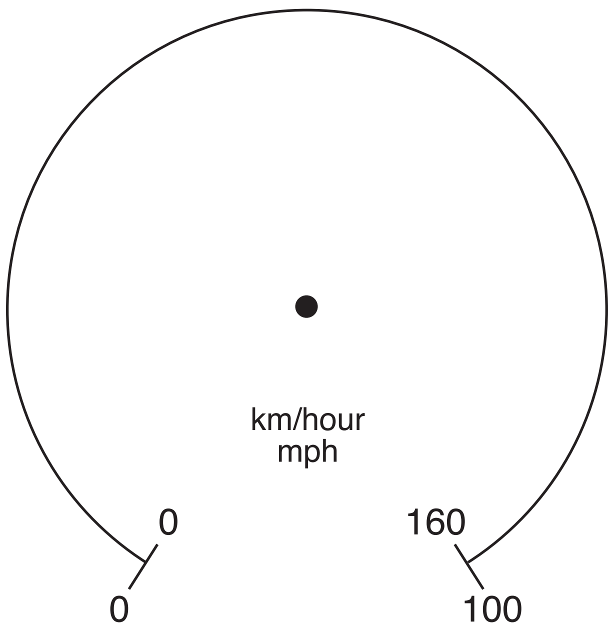
2. Mark the fuel gauge to show 0.25, 0.5 and 0.75 full.

Estimate the fraction shown by the pointer.

__________ full

Mark the temperature gauge to show 10° intervals.

Draw the pointer when it is at 105°C.



3. Complete the scales for the speedometer dial on the left. The outside scale has to be marked from 0 to 100 mph at 10 mph intervals. The inside scale has to be marked from 0 to 160 at 10 km/hour intervals.

Draw the pointer of the speedometer when it is showing a speed of 60 mph.

What is 60 mph in km/hour?

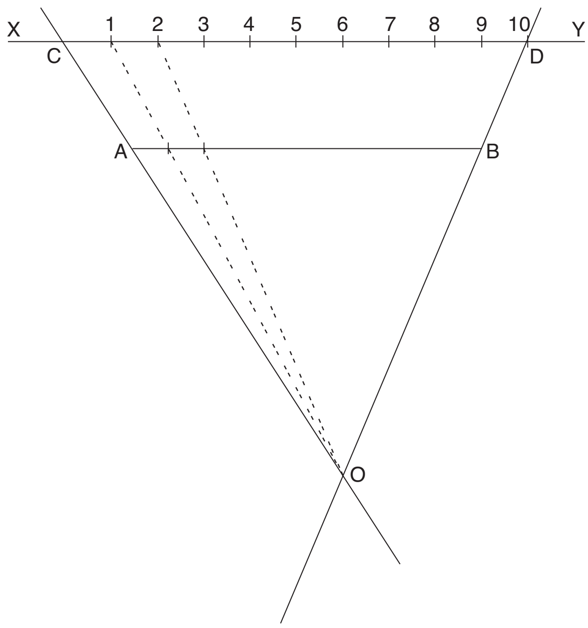
_________ km/hour

Dividing a Line by Intersection

Our problem is to divide a straight line AB into a number of equal parts, say ten. Why can't you just use a ruler? Sometimes you can. For example, if the length of AB is 5 cm, you can simply make a mark every 5/10 = 0.5 cm with your ruler.

However, suppose the length of the line is 7.75 cm. You now need to mark every 7.75/10 = 0.775 cm. This is much more difficult to do accurately.

There are two ways of constructing the divisions accurately. One is given on this sheet. (The other is dividing the line by parallels and is covered on the next sheet.)



Method

1. Draw a line XY parallel to AB It should be longer than AB and a few cm away - see diagram.
2. Use a ruler to measure accurately 10 equal spaces (for example, use 1 cm or 0.5 cm spacing). Mark this line CD. CD should be a little longer than AB.
3. Join C to A and D to B, and extend these lines to intersect at O - see diagram.
4. Now join up each division on CD to O. Mark clearly where these lines cross AB. These points give accurate divisions of the line into 10 equal parts.

Problems

1. Draw a line AB about 11 cm long, on a sheet of A4 paper. Use the method described above to divide AB into five equal divisions.
2. A line 6 cm long is to be calibrated for temperatures of between 45°C to 70°C. Use the method above to show the temperature every 5°C.

Extensions

(a) Will the method work if the line XY is shorter than AB?

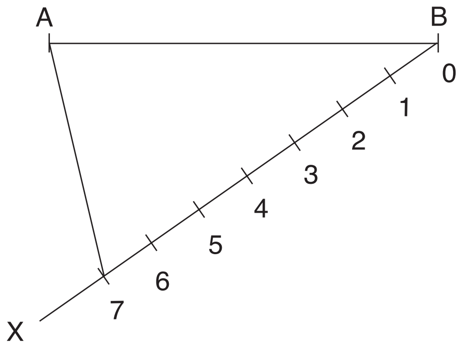
(b) Will the method work if XY is not drawn parallel to AB?

DIVIDING A LINE BY PARALLELS

Our problem is to divide a straight line AB into a number of equal parts, say seven. Why can't you just use a ruler? Sometimes you can. For example, if the length of AB is 5 cm, you can make a mark every 3.5/7 = 0.5 cm with your ruler.

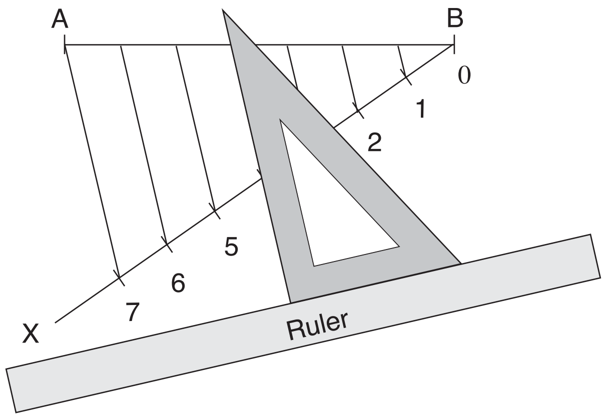
However, suppose the length of the line is 5.81 cm. You now need to mark every 5.81/7 = 0.03 cm. This is much more difficult to do accurately.

There are two ways of constructing the divisions accurately. One is given on this sheet. (The other is dividing the line by intersection shown earlier.)



METHOD

1. Draw a second line XB at an angle of about 60° to AB.
2. Mark seven equal divisions along BX - use a convenient size such as 1 cm.
3. Draw a line from A to the last division (marked 7).
4. Using a ruler and set-square, as shown opposite, draw lines parallel to A7 which pass through 6, 5, 4, 3, 2 and 1 on BX.
5. The intersection of these lines on AB divides it into seven equal parts.



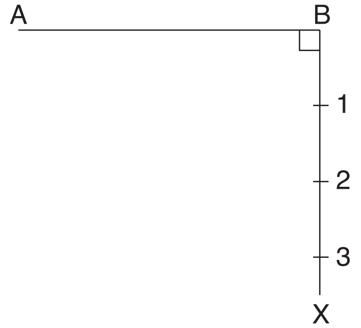
PROBLEMS

1. Use this method to divide a line of about 11 cm into six equal divisions.
2. A line 7 cm long is to be calibrated from 0 km per hour to 150 km per hour, showing every 10 km per hour. Use the method above to achieve this calibration.

EXTENSIONS

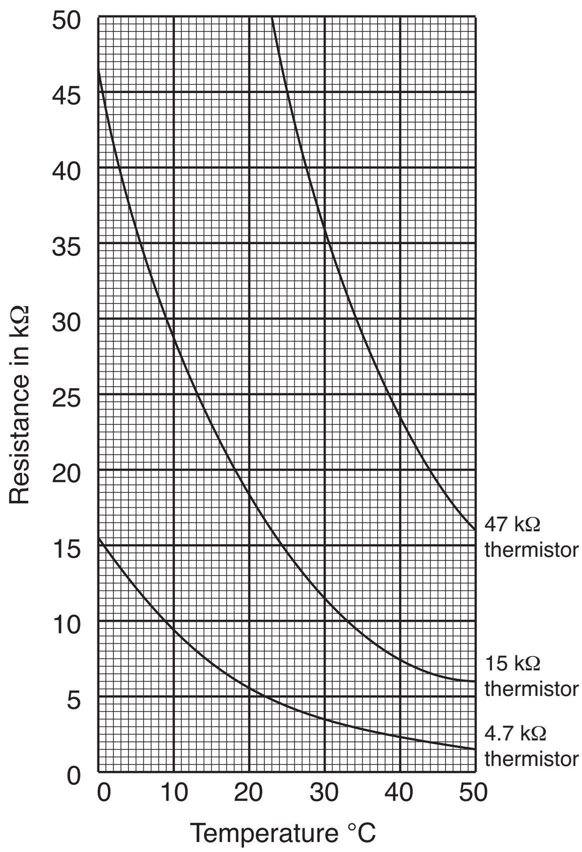
(a) What happens when the angle ABX is increased to 90° (see right)?

(b) Will this method still work if an angle of more than 90° is used?



READING A GRAPH

Look at the graph on thr right. You can use graphs to illustrate general trends. The divisions on the horizontal scale are numbered 0, 10, 20, ... etc, going up in tens. The divisions on the vertical scale are numbered 0, 5, 10, 15, ..., going up in fives. A larger version of this graph is given overleaf.



PROBLEMS

Use the graph to answer these questions:

1. As the temperature increases, what happens to the resistance of the thermistor?
2. If the resistance of the thermistor increases, what must have happened to the temperature?
3. On the horizontal scale, what does one small division represent?
4. On the vertical scale:

   (a) How many small divisions are needed to show 1 kΩ on the scale?

   (b) What does one small division represent?

   You can use this graph to make accurate readings. For example, for the 15 kΩ thermistor, at 10°C, its resistance is 29 kΩ.

5. Complete the table below, giving your answers in kΩ to the nearest 0.5.

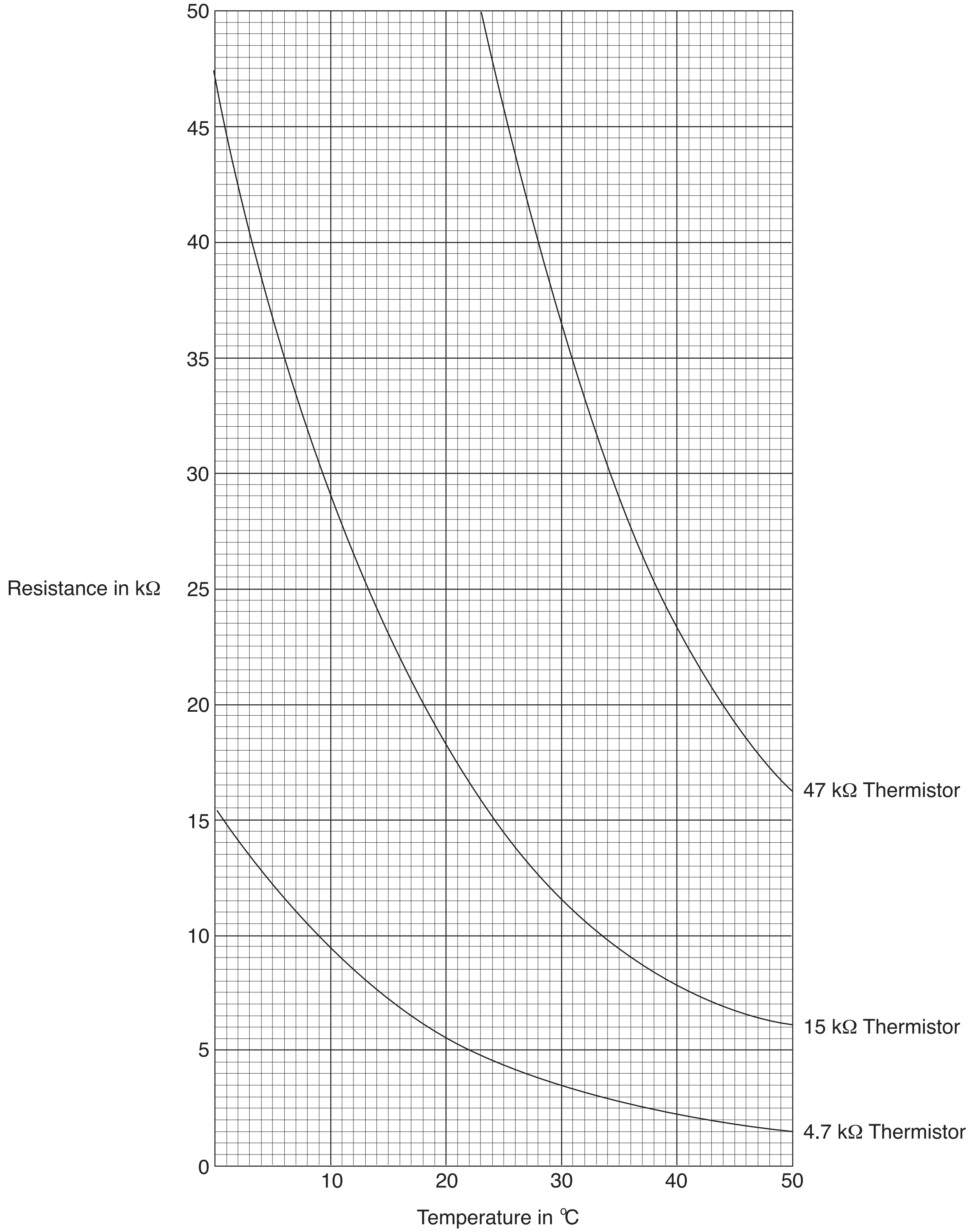
	Temperature °C								
	0	**5**	**10**	**15**	**20**	**25**	**30**	**40**	**50**
Resistance of: 4.7 kΩ thermistor			9.5				3.5		
15 kΩ thermistor			29.0				11.5		
47 kΩ thermistor							36.5		

6. For each thermistor find, if possible, the range of the resistance for:

   (a) 25°C to 45°C.
   (b) 0°C to 20°C.
   (c) 30°C to 40°C.

Extension
For each thermistor, find at what temperature it gives a resistance equal to that of its name. Suggest a rule for naming the thermistor.

7. This graph shows how the resistances of three thermistors varies with changes in temperature. What is the resistance of the 15 kΩ thermistor at temperatures of (a) 10°C and (b) 24°C?



NON LINEAR SCALES

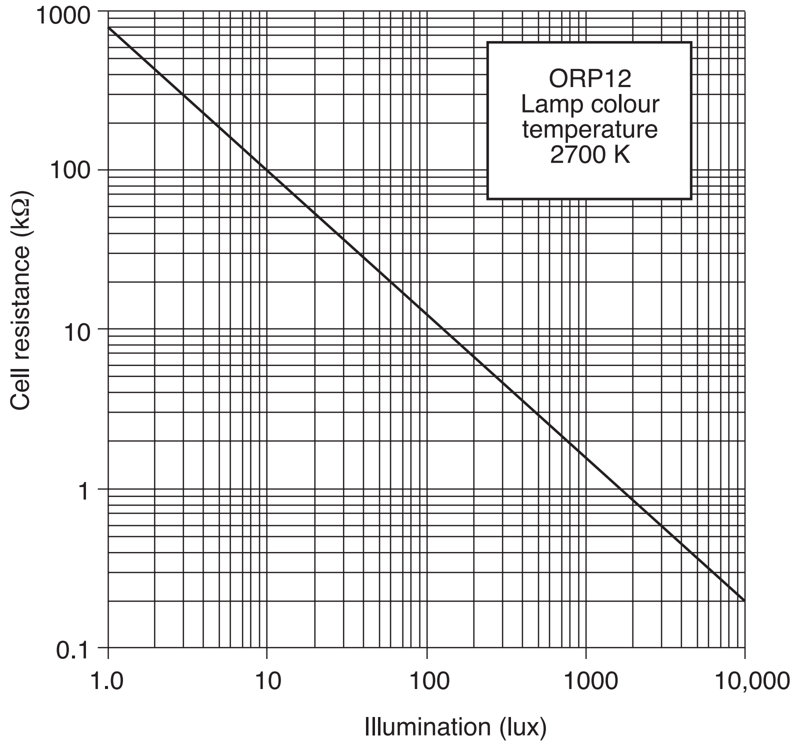
The graph on the right shows the relationship between

Cell resistance (kΩ)
and
Illumination (lux)

Although the graph line is straight, the relationship is not as simple as it looks because the scales on both axes are not evenly spaced. They are **non-linear scales**.

Look at the vertical axis and count the number of lines marked between 1 and 10. They correspond to 2, 3, 4,.., 9.

Similarly, from 10 to 100, the lines correspond to 20, 30, 40, ..., 90.



PROBLEMS

1. What do the lines (on the vertical scale) between 100 and 1000 represent?
2. What do the lines between 0.1 and 1 represent?

   The horizontal scale is also non-linear and follows a similar pattern to the vertical scale. We are now ready to read off values from the graph. For example, an illumination of 60 lux corresponds to 20 kΩ.
3. Estimate the resistance that corresponds to an illumination of:

   (a) 2 lux (b) 20 lux (c) 500 lux (d) 100 lux
4. Estimate the illumination that corresponds to a resistance of:

   (a) 2 kΩ (b) 50 kΩ (c) 0.3 kΩ (d) 10 kΩ

Extension
The equation represented by the graph is given by the formula

$$y=800x^{-0.9}$$

where y is the cell resistance (kΩ) and x the illumination (lux). Check the values of y from the graph and formula when

x=1, 10, 100, 1000, 10 000.

Solutions

Prefixes

Problems

1. (a) 7 k watts (7 kW)
   (b) 8 M volts (8 MV)
   (c) 5.5 k ohms (5.5 kΩ)
   (d) 4 G watts (4 GW)

2. (a) 7 kΩ
   (b) 3.4 kΩ
   (c) 19.7 kΩ
   (d) 5.25 kW
   (or 5250 W)
   (e) 6507 kV
   (or 6.507 MV)
   (f) 21 kΩ

Extension

(a) 1 m metre (or 1 mm)
(b) 5 μ second (or 5μs)
(c) 5 m litre (or 5 ml)

Measures and Errors

Problems

1. (a) 0.23 cm (or 2.3 mm)
   (b) 1.5 cm (or 15 mm)

2. (a) 40°C
   (b) 0.34 cm = 3.4 mm
   (c) (i) 1.6 cm too small
   (ii) 2.4 cm too big

3. (a) 11.3/35 =0.322 cm
   =3.2 mm

   (b) Using 3 mm, the error will be 0.8 cm.

   (c) Since the same number of divisions have to be made, the total error will still be 0.8 cm.

Dividing an Arc and Calibration

Problems

1.

Reading	Angle
0	0°
5	18°
10	36°
15	55°
20	73°
25	91°
30	109°
35	128°
40	146°
45	164°
50	182°
55	200°
60	219°
65	237°
70	255°

3. 63

4.

Reading	Angle
0	0°
10	22°
20	45°
30	67°
40	89°
50	112°
60	134°
70	156°
80	178°
90	201°
100	223°
110	245°
120	268°
130	290°

Extension

By the time the calibration is complete, you would have used 14 × 18 = 252° and you would have been 3° short at the far end point.

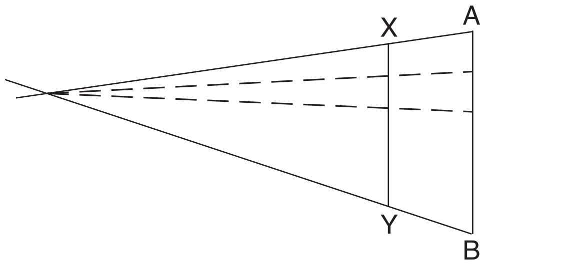
Calibration Questions

1. 187 volts
2. 5/8 full
3. 96 km/hour

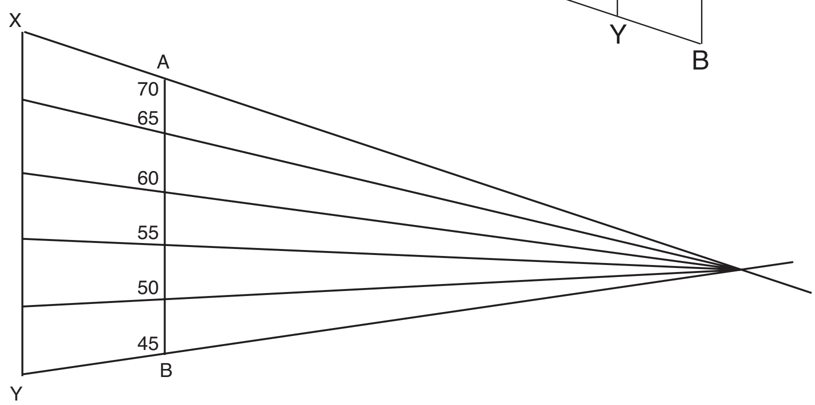
DIVIDING A LINE BY INTERSECTION

Problems

1.



2.



Extensions

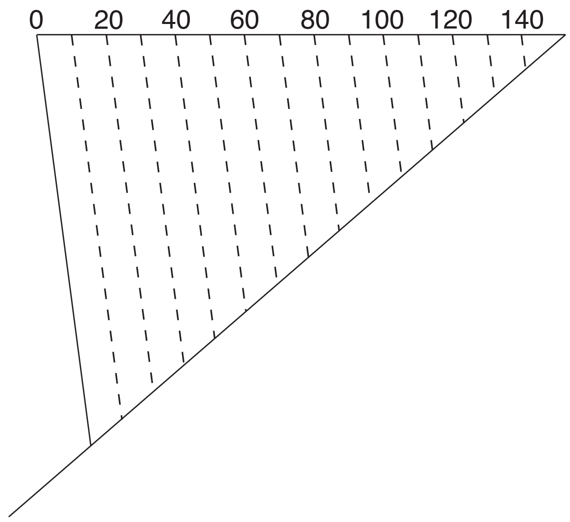
(a) Yes, but probably not so accurately.

(b) No! Divisions will not be equal unless lines are parallel.

DIVIDING A LINE BY PARALLELS

Problems

2.



Extensions

Yes, it would work for 90° or even angles larger than 90°, but it would become more liable to inaccuracies as the angle gets larger. It would not work for an angle of 180°.

READING A GRAPH

Problems

1. decreases.
2. decreases.
3. 1°C
4. (a) Two (b) 0.5 kΩ

5.

	0	**5**	**10**	**15**	**20**	**25**	**30**	**40**	**50**
4.7 kΩ	15.5	12.2	9.5	7.2	5.5	4.3	3.5	2.3	1.5
15 kΩ	47.0	36.5	29.0	23.0	18.2	14.4	11.5	7.8	6.1
47 kΩ	–	–	–	–	–	45.7	36.5	23.3	16.2

6.

	(a)	**(b)**	**(c)**
4.7k Ω	2.5	10.0	1.2
15 kΩ	6.7	28.8	3.7
47 kΩ	26.7	–	13.2

Extension

23.5°C 24°C 24.3°C Resistance at 24°C

7. (a) 29 kΩ (b) 15 kΩ

NON-LINEAR SCALES

Problems

1. 200, 300, 400, ..., 900
2. 0.2, 0.3, 0.4, ..., 0.9
3. (a) 400 kΩ (b) 50 kΩ (c) 3 kΩ (d) about 12 kΩ
4. (a) 750 lux (b) 20 lux (c) 6000 lux (d) about 120 lux

Extension

There should be a close fit between values. By calculation, you obtain

x	1	10	100	1000	10 000
y	800	101	12.7	1.6	0.2