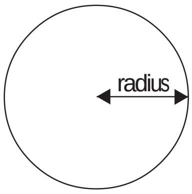
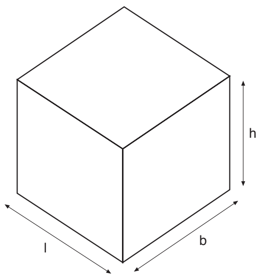
MATHS HELP

Technology uses mathematics in many different ways - ranging from simple measurement 'sums' in marking out materials to complicated calculations in engineering. Unfortunately, many people think of maths as very difficult and try to avoid using it when they are designing things. In fact, a lot of the maths needed in design work is really quite simple arithmetic and is a friend not an enemy! The following pages give you some examples of how maths can help in your design work.

Areas of shapes tells you how to work out the areas of a number of common shapes. For example, you may need to work out the cost of a circular disc of material which is priced in pence per square cm (e.g. silver). On a calculator, you simply have to multiply the radius of the blank by itself (r^2) and then multiply this figure by π which on most calculators means pressing the π key.



Volumes of shapes tells you how to work out the volumes of a number of different 'containers'. It is only slightly more complicated than working out areas. For example, the volume of a cylinder = the area of the base (identical to the calculation on the area of a disc) × the length. This simple calculation is often used for estimating how much liquid a container will hold. With a bit of thought, you can work out the dimensions of different container sizes needed to hold a specified amount of liquid. For example, you make an assumption about (guess) the diameter of the base and divide its area into the volume of liquid to give the length.



It is important to note that if you are estimating the volume of a complicated container shape, you can break it down into a number of simpler shapes - e.g., a cylinder with a hemispherical dome at both ends has a volume of the cylinder + a sphere.

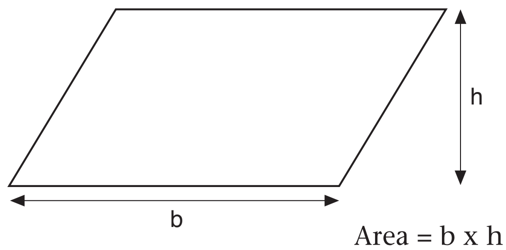
If you know the volume of a piece of material - e.g., your design for a paperweight - you can predict its overall mass by multiplying the volume by *density*.

AREAS OF SHAPES

Rectangle

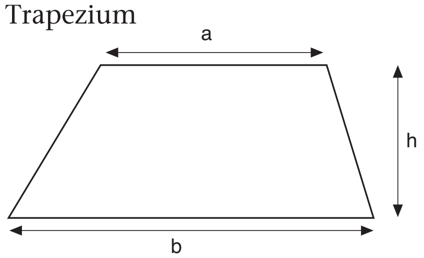
Area = l x b

Parallelogram



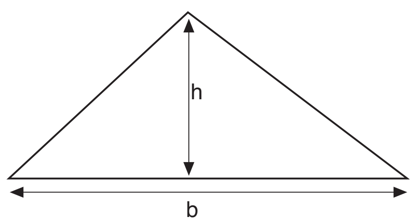
Area = b x h

Trapezium



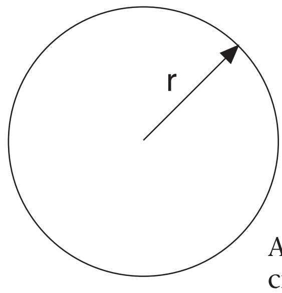
Area = $\frac{(a+b)h}{2}$

Triangle



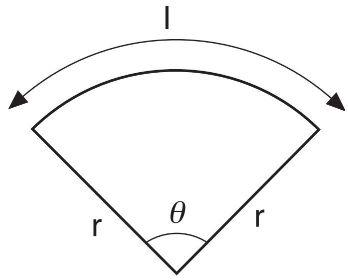
Area = $\frac{bh}{2}$

Circle



Area = πr^2
circumference = $2\pi r$

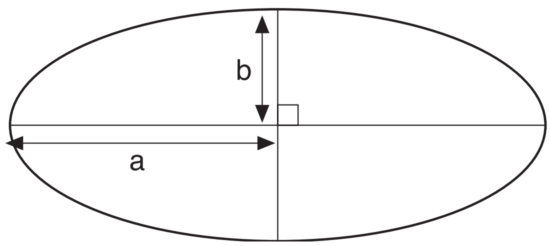
Sector of a circle



Area = $\frac{\theta}{2\pi} \times \pi r^2 = \frac{r^2 \theta}{2}$ (θ in rad.)*

Length of arc (l) = $\frac{\theta}{2\pi} \times 2\pi r = r\theta$ (θ in rad.)*

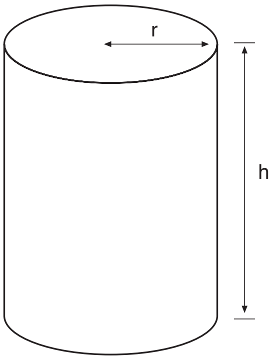
Ellipse



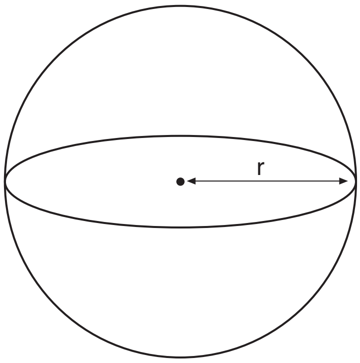
Area = πab
Perimeter = $1.414\pi\sqrt{(a^2+b^2)}$ (approx.)

* (θ in radians)
1 radian = $\frac{360}{2\pi}$ = 57.3
1° = 0.0175 radian

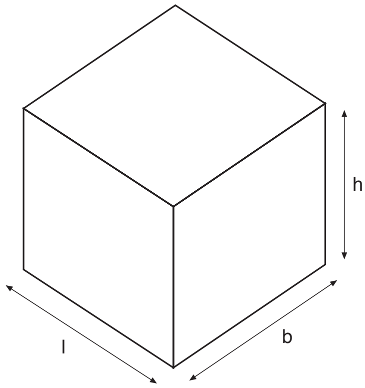
VOLUMES OF OBJECTS



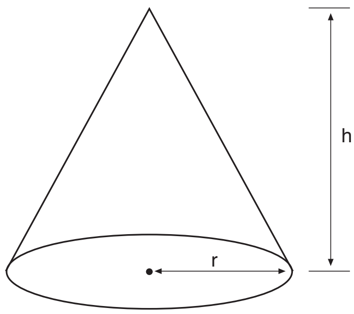
Volume of cylinder = $\pi r^2 \times h$



Volume of sphere = $\frac{4\pi r^3}{3}$



Volume of box = $l \times b \times h$

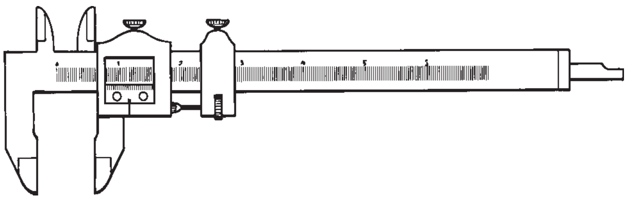


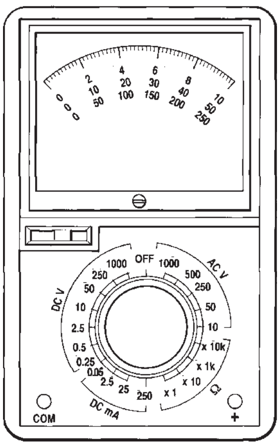
Volume of cone = $\frac{\pi r^2 h}{3}$

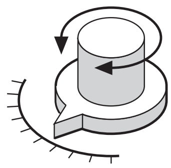
Dividing and Calibrating

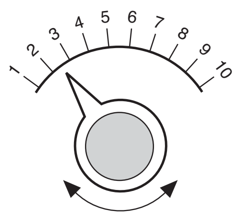
There is often a need in technology to divide lines and circles into equal divisions with great accuracy. For example, you might be marking out a piece of work to drill a series of equally spaced holes or putting a series of numbers on a clock dial. The next few pages give you some methods for dividing and calibrating. They also provide some problems to practise on.

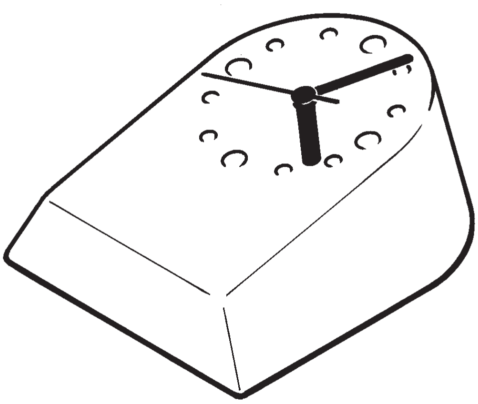
Examples of calibration









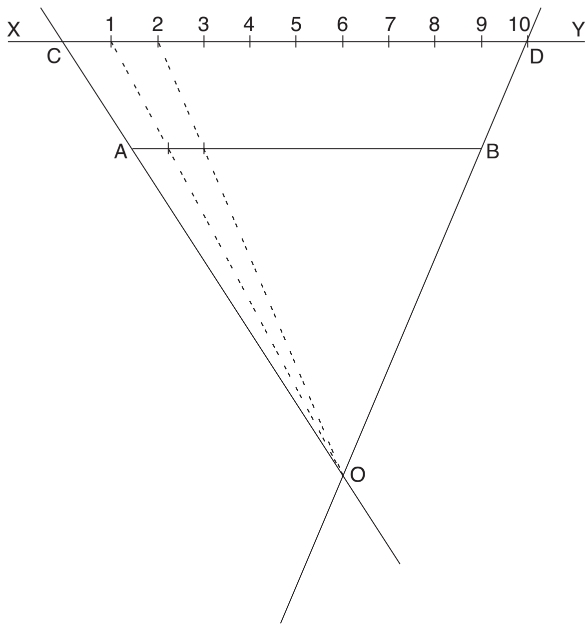


DIVIDING A LINE BY INTERSECTION

Our problem is to divide a straight line AB into a number of equal parts, say ten. Why can't you just use a ruler? Sometimes you can. For example, if the length of AB is 5 cm, you can simply make a mark every 5/10 = 0.5 cm with your ruler.

However, suppose the length of the line is 7.75 cm. You now need to mark every 7.75/10 = 0.775 cm. This is much more difficult to do accurately.

There are two ways of constructing the divisions accurately. One is given on this sheet. (The other is dividing the line by parallels and is covered on the next sheet.)



METHOD

1. Draw a line XY parallel to AB It should be longer than AB and a few cm away - see diagram.
2. Use a ruler to measure accurately 10 equal spaces (for example, use 1 cm or 0.5 cm spacing). Mark this line CD. CD should be a little longer than AB.
3. Join C to A and D to B, and extend these lines to intersect at O - see diagram.
4. Now join up each division on CD to O. Mark clearly where these lines cross AB. These points give accurate divisions of the line into 10 equal parts.

PROBLEMS

1. Draw a line AB about 11 cm long, on a sheet of A4 paper. Use the method described above to divide AB into five equal divisions.
2. A line 6 cm long is to be calibrated for temperatures of between 45°C to 70°C. Use the method above to show the temperature every 5°C.

EXTENSIONS

(a) Will the method work if the line XY is shorter than AB?

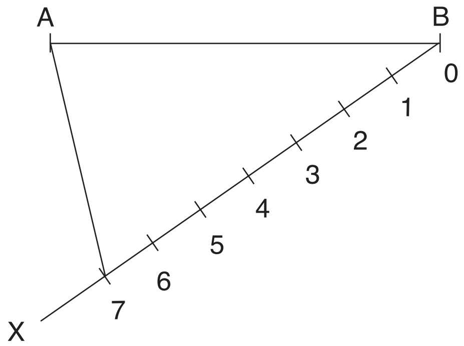
(b) Will the method work if XY is not drawn parallel to AB?

DIVIDING A LINE BY PARALLELS

Our problem is to divide a straight line AB into a number of equal parts, say seven. Why can't you just use a ruler? Sometimes you can. For example, if the length of AB is 3.5 cm, you can make a mark every 3.5/7 = 0.5 cm with your ruler.

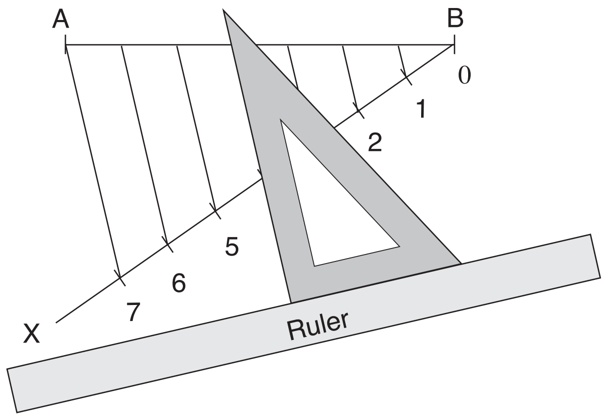
However, suppose the length of the line is 5.81 cm. You now need to mark every 5.81/7 = 0.83 cm. This is much more difficult to do accurately.

There are two ways of constructing the divisions accurately. One is given on this sheet. (The other is dividing the line by intersection shown earlier.)



METHOD

1. Draw a second line XB at an angle of about 40° to AB.
2. Mark seven equal divisions along BX - use a convenient size such as 1 cm.
3. Draw a line from A to the last division (marked 7).
4. Using a ruler and set-square, as shown opposite, draw lines parallel to A7 which pass through 6, 5, 4, 3, 2 and 1 on BX.
5. The intersection of these lines on AB divides it into seven equal parts.



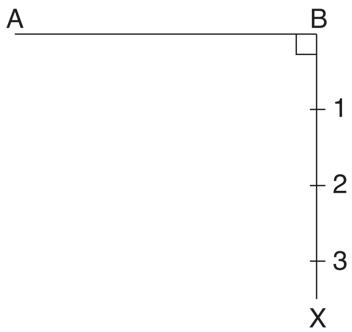
PROBLEMS

1. Use this method to divide a line of about 11 cm into six equal divisions.
2. A line 7 cm long is to be calibrated from 0 km per hour to 150 km per hour, showing every 10 km per hour. Use the method above to achieve this calibration.

EXTENSIONS

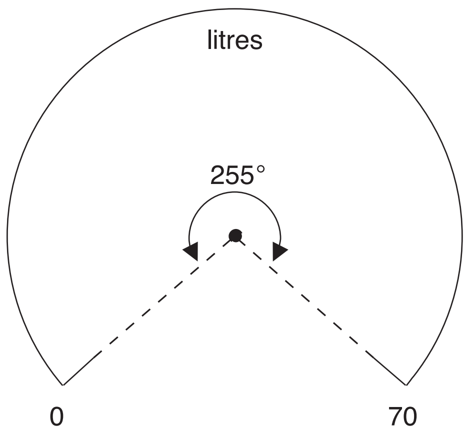
(a) What happens when the angle ABX is increased to 90° (see right)?

(b) Will this method still work if an angle of more than 90° is used?



DIVIDING AN ARC AND CALIBRATION

Our problem here is to calibrate a rotary control knob. For example, suppose you want to read the volume of liquid in a tank in litres. The lowest value is 0 litres and the largest value 70 litres. The knob turns through 255°.



You want to read every 5 litres on the scale; 0, 5, 10, 15, ..., 65, 70. Since 70 litres is represented by an angle of 255°, 1 litre is represented by

$$\frac{255°}{70} = 3.6428°$$

and 5 litres is represented by

$$5 \times \frac{255°}{70} = 18.214°$$

which is 18° to the nearest degree.

You can continue in this way:

10 litres needs an angle of

$$10 \times \frac{255°}{70} = 36.428°$$

which is 36° to the nearest degree.

15 litres needs an angle of

$$15 \times \frac{255°}{70} = 54.642°$$

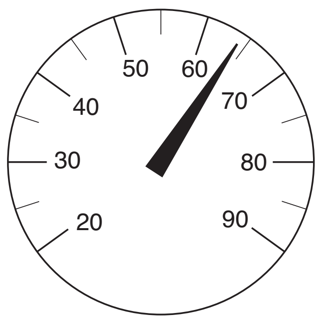
which is 55° to the nearest degree.

PROBLEMS

1. Copy and complete the table below to give all the required angles:

Reading (litres)	**Angle** (nearest degree)
0	0°
5	18°
10	36°
20	55°
25	-
30	-
35	-
40	-
45	-
50	-
55	-
60	-
65	-
70	255°

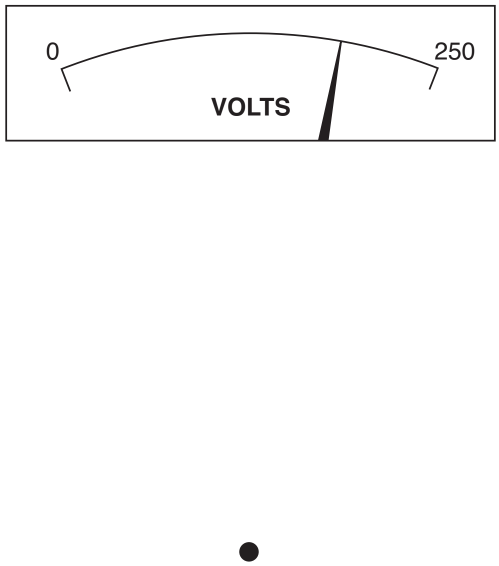
2. Using a circle of radius 4 cm, calibrate a 255° sector of a circle to show 0 to 70 litres in 5 litre divisions.
3. A completed calibration from 20 to 90 is shown in the diagram below. Estimate the reading shown by the pointer.



4. A similar dial has to be calibrated from 0 to 130 litres, marked off in 10 litre intervals. The total angle to be used is 290°. Complete a table of angle calculations and draw the diagram.

EXTENSION

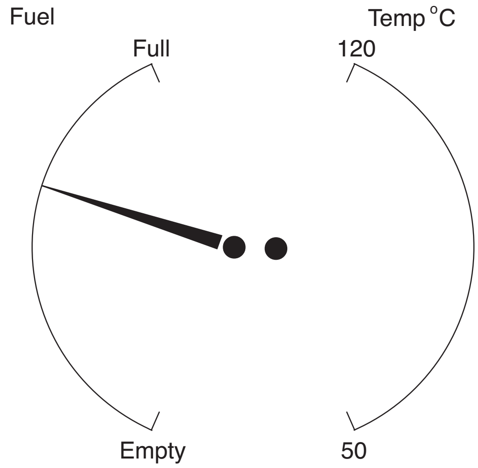
Suppose you use 18° for each 5 litre division in the first problem. Would your calibration be accurate enough?



1. Complete the marking of the voltmeter scale to show 50, 100, 150 and 200 volts.

What voltage is being shown by the pointer?

_______ volts



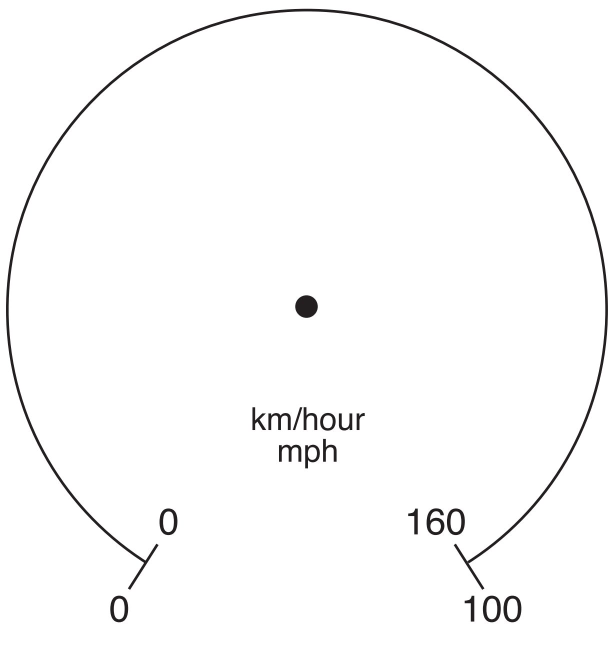
2. Mark the fuel gauge to show 0.25, 0.5 and 0.75 full.

Estimate the fraction shown by the pointer.

__________ full

Mark the temperature gauge to show 10° intervals.

Draw the pointer when it is at 105°C.



3. Complete the scales for the speedometer dial on the left. The outside scale has to be marked from 0 to 100 mph at 10 mph intervals. The inside scale has to be marked from 0 to 160 at 10 km/hour intervals.

Draw the pointer of the speedometer when it is showing a speed of 60 mph.

What is 60 mph in km/hour?

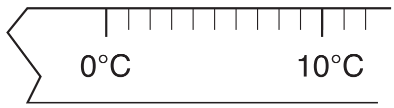
_________ km/hour

MEASURES AND ERRORS

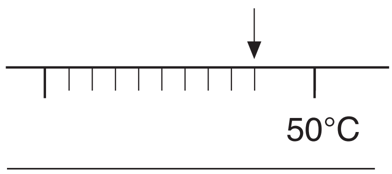
You are making a thermometer scale. It is a straight line scale, and all the divisions on it must be equally spaced. The positions of the lowest and highest temperatures, (0°C and 50°C) are 10.5 cm or 105 mm apart.

So, a change in temperature of 50°C is shown by the length 105 mm. This means a change of 1°C is shown by the length 105/50 = 2.1 mm. As it is difficult to measure 2.1 mm, you might decide to use 2 mm which is close.

Laying your ruler along the scale and, starting at 0, you make a mark every 2 mm.



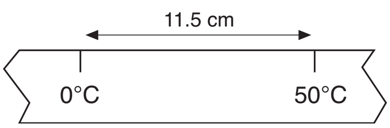
Finally, you would make the 50th mark as shown but it is not where it should be at the 50°C mark.



So what went wrong? Fifty spaces at 2 mm each is 50×2 = 100 mm but 105 mm were actually needed. Though 2 mm seems close to 2.1 mm, when used several times over it can make a big error in the end. The total error is 105 - 100 = 5 mm too small.

PROBLEMS

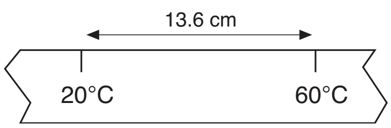
1. Sara is making a thermometer to measure from 0°C to 50°C. On her scale the distance between the two is 11.5 cm.



(a) What should the distance be between each degree?

(b) Sara uses 2 mm spacing to mark the degrees. What will her total error be at 50°C?

2. Mark is making a thermometer to show 20°C to 60°C. The measured distance between those points is 13.6 cm.



(a) How many degrees does Mark's thermometer show?

(b) What should the measured distance be for each degree?

(c) He marks the degree spaces starting from 20°C. What will his total error be if he uses

(i) 3 mm for each degree?
(ii) 4 mm for each degree?

3. Rohan is making an electronic weighing machine to weigh between 100 and 800 grams only. The scale is a straight line and, between those two points, the distance is 11.3 cm. Rohan wants to put a mark on the scale at every 20 grams.

(a) What distance is needed between each mark?

(b) Working to the nearest millimetre, what error will Rohan have at the end?

(c) If Rohan measured the marks in from both ends, what would his error be when he reached the middle?