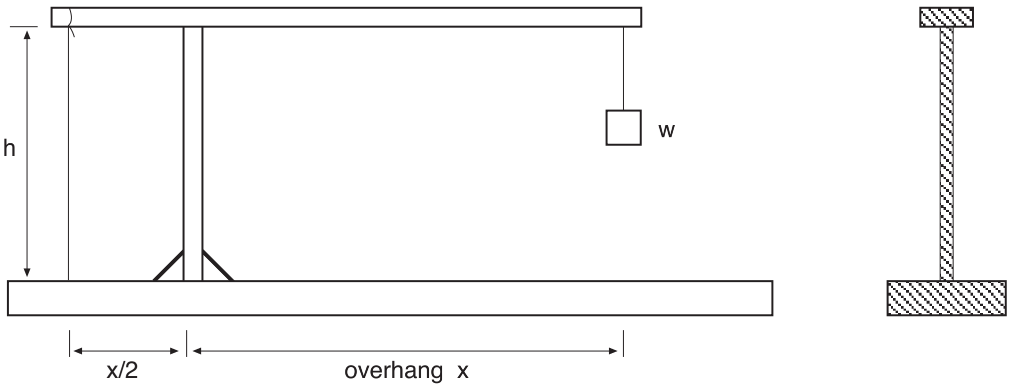
EXPERIMENTAL TOWER

Design, build and test a simple, three element structure as shown below. This form of structure could be used for many purposes. For example, depending on its size, it could be the support for a desktop reading light, the boom support for an overhead microphone or it could form the basis of a display stand. In studying this structure you will also gain an understanding of the principles of many large structures such as the tower cranes used on building sites and docksides.



The structure is made of:

- *A lightweight beam of hardwood strip of cross section about 20 × 10 mm*
- *A supporting vertical strut of plastic rod*
- *A backstay of several turns of thin copper or steel wire*

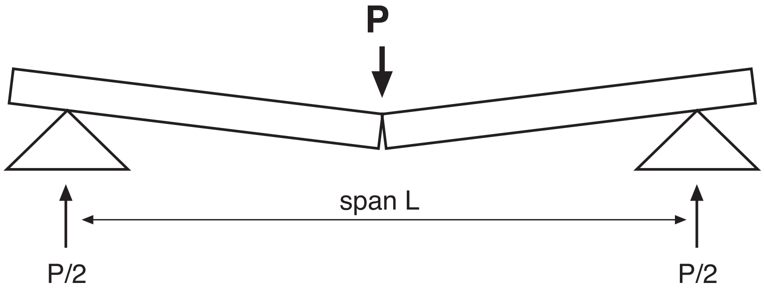
Your task is to choose the length of overhang x of the beam, the height h of the strut and the number of turns of wire such that the model will carry a load of 2kg without failing, but when a load of 3kg is applied all 3 components should have failed.

DESIGN

Your design will consist of a mixture of practical measurements and calculations.

1) Your first step is to measure the ability of the hardwood beam to withstand bending without breaking. This is the **ultimate moment capacity**, $\mathbf{M_{ult}}$ of the beam. It is not easy to predict this accurately for a timber beam, because different timbers have different ultimate tensile strengths. Even strips of timber from the same tree will show a variation in strength.

On a separate sample of the hardwood strip, carry out a three point bend test to failure by supporting the strip close to its ends and increasing the load, **P**, at the centre in small steps until the strip breaks.



The **ultimate moment** can then be found from the formula,

$$M_{ult} = \frac{P}{2} \times \frac{L}{2} = \frac{PL}{4}$$

where **P** is the breaking load found in the test.

(For interest test the beam about its stronger axis also).

Now you can find the distance of maximum overhang, x, for the model. The maximum moment that the beam must support in the model is at the support and is given by,

$$\mathbf{M_{ult} = W.x}$$

where **W** is 4kg wt or 39.2 N.

2) The backstay support is made of several turns of thin copper or steel wire. Measure the ultimate tensile strength of this wire by hanging a gradually increasing weight on the end of a single strand until it breaks.

Now you need to use the **principle of moments** to calculate the tension force, **T**, which the backstay will have to carry to balance the load at the other end of the beam.

When the beam is in equilibrium (i.e. when its balanced),

$$\frac{Tx}{2} + \frac{B}{3} \cdot \frac{x}{4} = Wx + \frac{2B}{3} \cdot \frac{x}{2}$$

(where B = weight of the beam).

$$\frac{Tx}{2} = Wx + \frac{Bx}{4}$$

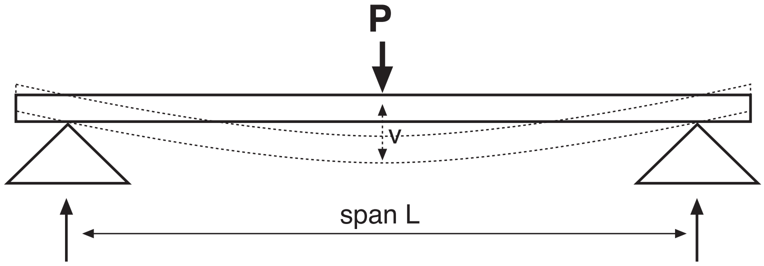
$$T = 2W + \frac{B}{2}$$

From the value of the tension force you can now decide how many turns of wire are needed for your crane model.

3) The column of the crane is a slender strut loaded in compression. As discussed earlier in the unit this will fail by buckling when the load carried is high enough. This load, **F**, is simply the sum of the vertical forces acting on the beam, i. e.

F = W + T + weight of beam (B)

Next, as a preliminary test for the properties of the strut, do a three point bend test like you did for the beam, but this time not to failure. Increase the load on the mid point of the member in steps and measure the mid span deflection, **v**. Plot a graph of **P** against **v**. You should be able to fit a good straight line to the points on your graph.



The equation for the mid span deflection is,

$$v = \frac{PL^3}{48EI} \quad \text{or} \quad P = \frac{48EIv}{L^3}$$

You can now find the value of the term **EI** from the gradient of the graph. This is called the **flexural stiffness** and is needed for the next step.

For a slender strut clamped at its base and free at its head, the load at which it will buckle is given by,

$$F = \pi^2 \frac{EI}{4L^2}$$

Now you can find the height of your crane, **h**, by setting **h = L** in the above equation with **F** as the load.

BUILDING THE MODEL

This is quite straightforward, except that it is necessary to fix the base of the strut very firmly so that the crane doesn't topple. You will need to devise some kind of clamp arrangement to achieve this.

TESTING

Starting with a load of 1 or 2 kg, increase its size in steps until one of the components breaks. Note this load and how the component has failed. Now repair the model by replacing the failed component with a much stronger substitute (of the same length). Continue testing until the second component fails. Replace this with a stronger substitute and continue to increase the load until the third element fails.

Calculate the safety factor of each of the three components.

INVESTIGATING THE EFFECTS OF DIFFERENT CROSS SECTIONS

We mentioned earlier that the cross section of the strut has an effect on the load at which it buckles. This is because the shape of the cross section affects the stiffness of the strut. You can demonstrate this very simply by bending a plastic ruler. It's much stiffer in one direction than the other. It is also quite easy to show that hollow tubes are much stiffer than solid rods of the same mass and that the bigger the diameter of a tube, the stiffer it is. In other words it's not just the amount of material in a strut that determines how stiff it is, but how this material is distributed in the cross section.

In rough terms, the further the material is placed from the centre of the struts cross section, the stiffer it will be. What is true for struts (resisting compression) is also true for beams (resisting bending). Beams are normally designed to carry loads in a particular direction and are therefore often deliberately designed to be stiffer in one direction than another. The simplest example of this is the use of a rectangular cross section for timber floor joists.

In mathematical terms, the second moment of area (I) of the cross section is the measure of the distribution of the material in a cross section which can be used to determine the stiffness of a beam or strut. The bigger this is, the stiffer the part.

For the final part of your investigation of the tower crane structure you are asked to choose more suitable cross sections for the strut and the beam, ***keeping the lengths of the parts the same as before and as close as possible to the same weight as you used in your previous model.***

Look at some photographs of structures and at some local buildings to help you decide on the best cross sections to use.

Carry out the load deflection tests described above and find the value of EI for your new elements.

Test the model by loading it until one element fails. Compare its performance with your first model by working out the value of,

max load / weight of structure

for each structure.

You should find that your new design has a higher load / weight ratio.